BICRITERIA IN N×2 FLOW SHOP SCHEDULING UNDER SPECIFIED RENTAL POLICY WITH UNCERTAIN PROCESSING TIME

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Abstract

This paper is an attempt to obtain an optimal solution for optimizing the bicriteria taken as minimization of the total rental cost of the machines taken on rent under a specified rental policy subject to obtain the minimum makespan for n-jobs, 2-machine flowshop scheduling problem. The processing time of jobs are uncertain in nature that is not known exactly and are in fuzzy environment. The fuzzy processing times are described by triangular fuzzy membership function. Further the concept of job blockage in processing of jobs is introduced. A numerical example is provided to demonstrate the computational efficiency of proposed algorithm.

Keywords: Flowshop scheduling, Fuzzy Processing Time, Rental Cost, Average High Ranking, Utilization Time and Job Block.

Mathematical Subject Classification: 90B30, 90B35.

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1. Introduction

The scheduling of jobs and the control of their flow through a production process is essential to modern production/manufacturing companies. Ever since the first results of modern scheduling theory appeared some 50 years ago, scheduling has attracted a lot of attention from both academia and industry. Deterministic production scheduling models and algorithms assume that all parameters are well known and precisely defined. However, as almost all real-world production systems are fraught with uncertainities, and this prevents the application of deterministic scheduling theory. Information about production/manufacturing processes can be both imprecise and/or incomplete, or sometimes does not exist. In this case, application of standard methods of probability theory becomes difficult and often inappropriate. Fuzzy sets provide an appropriate tool for handling imprecise information. As an attempt to bridge the existing gap between the scheduling theory and practice, fuzzy scheduling models that utilise multicriteria approaches which mainly focused on two criteria, also known as bicriteria have been developed and reported in the literature. The bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. A flow shop scheduling problem has been one of the classical problems in production scheduling since Johnson (1954) proposed the well known Johnson's rule in the two stage flow shop scheduling problem. Smith (1967) studied minimization of mean flow time and maximum tardiness. Van Wassenhove and Gelders (1980) studied minimization of maximum tardiness and mean flow time explicitly as objective. MacCahon and Lee (1990) discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee (1996) addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Some of the noteworthy approaches are due to Zadeh (1965), Gupta J.N.D (1975), Maggu and Das (1977), Yager (1981), Marin and Roberto (2001), Yao and Lin (2002), Singh and Gupta (2005), Singh, Sunita and Allawalia (2008).

Gupta Deepak et al. (2007) studied bicriteria in n jobs two machines flowshop scheduling under predefined rental policy. In the present work, we have developed a heuristic algorithm for bicriteria two stage flow shop scheduling in which processing times are not known exactly and only estimated values are given, .i.e. fuzzy in nature. Fuzzy techniques in the form of approximate reasoning provide decision support and expert systems with powerful reasoning capabilities.

2. Practical Situation

Fuzzy set theory has emerged as a profitable tool for controlling and steering of systems and complex industrial processes, as well as for household and entertainment electronics. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. Medical science can save the patient's life but proper care leads to a faster recovery. Care giving techniques often require hi-tech, expensive medical equipment. Many of these equipments can even help in saving the life of critical patients. Most of these equipments are expensive & they are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. Many patients even lose their lives just because they cannot afford to buy these products. In his starting career, we find a medical practitioner does not buy expensive machines. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allow upgradation to new technology. Further the priority of one job over the other may be significant due to the relative importance of the jobs. It may be because of urgency or demand of that particular job. Hence, the job block criteria become important.

3. Basic Fuzzy Concepts

3.1. Fuzzy Membership Function

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$, where a_1 and a_3 denote the lower and upper limits of support of a fuzzy set $\tilde{A}$. The membership value of the x denoted by $\mu(x), x \in R^+$, can be calculated according to the following formula.

$$\mu(x)=\begin{cases}0; x\le a_1\\ \dfrac{x-a_1}{a_2-a_1}; a_1<x<a_2\\ \dfrac{a_3-x}{a_3-a_2}; a_2<x<a_3\\ o; x\ge a_3\end{cases}$$

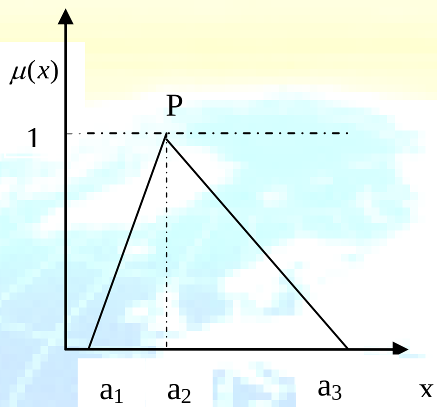

Figure 1: Triangular fuzzy number

3.2. Average High Ranking <A.H.R.>

To find the optimal sequence, the processing times of the jobs are calculated by using Yager's (1965) average high ranking formula (AHR) $= h(A) = \dfrac{3a_2 + a_3 - a_1}{3}$.

3.3. Fuzzy Arithmetic Operations

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

(i) $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_1})$

(ii) $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$ if the following condition is satisfied $DP\left(\tilde{A}_1\right) \ge DP\left(\tilde{A}_2\right)$, where $DP(\tilde{A}_1) = \dfrac{\beta_{A_1} - m_{A_1}}{2}$ and $DP(\tilde{A}_2) = \dfrac{\beta_{A_2} - m_{A_2}}{2}$. H, say ere DP denotes difference point of a Triangular fuzzy number.

Otherwise; $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - \beta_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - m_{A_2})$

(iii) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if $k>0$.

(iv) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if $k<0$.

(v) $A\times B=(\min(a_1b_1,a_1b_3,a_3b_1,a_3b_3),\max(a_1b_1,a_1b_3,a_3b_1,a_3b_3))$

(vi) $A/B=(\min(a_1/b_1,a_1/b_3,a_3/b_1,a_3/b_3),\max(a_1/b_1,a_1/b_3,a_3/b_1,a_3/b_3))$.

4. Notations

S : Sequence of jobs 1,2,3,….,n

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -----

M_j : Machine j, j= 1,2

M : Minimum makespan

a_{ij} : Fuzzy Processing time of i^{th} job on machine M_j

A_{ij} : AHR of processing time of i^{th} job on machine M_j

$L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j

t'_{ij} :Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$

$I_{ij}(S_k)$: Idle time of machine M_j for job *i* in the sequence S_k

$U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $L_j(S_k)$

$R(S_k)$:Total rental cost for the sequence S_k of all machine

C_i : Rental cost of i^{th} machine

$CT(S_i)$:Total completion time of the jobs for sequence S_i

4.1. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as :

$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + a_{ij}$ for $j\geq 2$.

where $a_{i,j}$ = Fuzzy processing time of i^{th} job on j^{th} machine

4.2. Definition

Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by t'_{ij} and is defined as

$$t'_{i,j}=L_j+\sum_{k=1}^{i}a_{k,j}=\sum_{k=1}^{i}I_{k,j}+\sum_{k=1}^{i}a_{k,j}$$

Also $t'_{i,j} = \max(t_{i,j-1}, t'_{i-1,j}) + a_{i,j}$.

4.3. Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine.

5. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the fuzzy processing time of i^{th} job on j^{th} machine which are described by triangular fuzzy numbers. Let A_{ij} ; i=1,2,3,.....n; j=1,2 be the average high ranking (AHR) of the processing times on two machines M_1 and M_2. Our aim is to find the sequence $\{S_k\}$of the jobs which minimize the rental cost of the machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1	Machine M_2
i	(a_{i1}, a_{i2}, a_{i3})	$(a'_{i1}, a'_{i2}, a'_{i3})$
1	(a_{11}, a_{12}, a_{13})	$(a'_{11}, a'_{12}, a'_{13})$
2	(a_{21}, a_{22}, a_{23})	$(a'_{21}, a'_{22}, a'_{23})$
-	-	-
n	(a_{n1}, a_{n2}, a_{n3})	$(a'_{n1}, a'_{n2}, a'_{n3})$

(Table 1)

Mathematically, the problem is stated as

Minimize $U_j(S_k)$ and

Minimize $R(S_k) = \sum_{i=1}^{n} a_{i1}(S_k) \times C_1 + U_2(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

6. Theorem

The processing of jobs on M_2 at time $L_2 = \sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof. Let $t'_{i,2}$ be the completion time of i [th] job on machine M_2 when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let P(n) : $t'_{n,2} = t_{n,2}$

Basic step: For n = 1, j =2;

$$t'_{1,2} = L_2 + \sum_{k=1}^{1} a_{k,2} = \sum_{k=1}^{1} I_{k,2} + \sum_{k=1}^{1} a_{k,2}$$

$$= \sum_{k=1}^{1} I_{k,2} + a_{1,2} = I_{1,2} + a_{1,2} = a_{1,1} + a_{1,2} = t_{1,2}$$

$\therefore$ P(1) is true.

Induction Step: Let P(m) be true, i.e., $t'_{m,2} = t_{m,2}$

Now we shall show that P(m+1) is also true, i.e., $t'_{m+1,2} = t_{m+1,2}$

Since $t'_{m+1,2} = \max(t_{m+1,1}, t'_{m,2}) + a_{m+1,2}$

$$= \max\left(t_{m+1,1}, L_2 + \sum_{i=1}^{m} a_{i,2} \right) + a_{m+1,2}$$

$$= \max\left(t_{m+1,1}, \left(\sum_{i=1}^{m} I_{i,2} + \sum_{i=1}^{m} a_{i,2} \right) + I_{m+1} \right) + a_{m+1,2}$$

$$t_{n,2} = L_2 + \sum_{i=1}^{n} a_{i,2} = \max\ t_{m+1,1}, t_{m,2} + I_{m+1} \ + a_{m+1,2}$$

$$= \max\ t_{m+1,1}, t'_{m,2} + \max\ \ t_{m+1,1} - t_{m,2}\ ,0 \ + a_{m+1,2} \qquad \text{(By Assumption)}$$

$$= \max\ t_{m+1,1}, t_{m,2}\ + a_{m+1,2}$$

$$= t_{m+1,2}$$

Therefore, *P(m+1)* is true whenever *P(m)* is true.

Hence by Principle of Mathematical Induction P(n) is true for all n i.e. $t'_{n,2} = t_{n,2}$ for all n.

Remark: If M_2 starts processing the job at $L_2 = t_{n,2} - \sum_{i=1}^{n} a_{i,2}$, then total time elapsed $t_{n,2}$ is not altered and M_2 is engaged for minimum time. If M_2 starts processing the jobs at time L_2 then it can be easily shown that $t_{n,2} = L_2 + \sum_{i=1}^{n} a_{i,2}$.

7. Algorithm

The following algorithm is proposed to minimize the rental cost of the machines taken on rent under a specified rental policy, when the processing time are uncertain in nature and represented by fuzzy triangular membership function.

Step 1: Find the average high ranking (AHR) A_{ij} ; i=1,2,3,...,n; j=1,2 of the processing times for all the jobs on two machines $M_1$and M_2.

Step 2: Take equivalent job β(*k*,*m*) and calculate the processing time $A_{\beta 1}$ and $A_{\beta 2}$ on the guide lines of Maggu and Das [6] as follows

$$A_{\beta 1} = A_{k1} + A_{m1} - \min(A_{m1}, A_{k2})$$

$$A_{\beta 2} = A_{k2} + A_{m2} - \min(A_{m1}, A_{k2})$$

Step 3: Define a new reduced problem with the processing time A_{ij} as defined in step 1 and jobs (k, m) are replaced by single equivalent job β with processing time $A_{\beta 1}$ and $A_{\beta 2}$ as derived in step 2.

Step 4: Using Johnson's technique [1] obtain all the sequences S_k having minimum elapsed time. Let these be S_1, S_2, ----------.

Step 5: Compute total elapsed time $t_{n,2}(S_k)$, k = 1,2,3,----, by preparing in-out tables for S_k.

Step 6: Compute $L_2(S_k)$ for each sequence S_k as follows

$$L_2(S_k) = t_{n,2}(S_k) - \sum_{i=1}^{n} a_{i,2}(S_k)$$

Step 7: Find utilization time of 2nd machine for each sequence S_k as

$$U_2(S_k) = t_{n,2}(S_k) - L_2(S_k)$$

Step 8: Find minimum of $\{U_2(S_k)\}; k = 1, 2, 3, ...$ Let it be for sequence S_p. Then S_p is the optimal sequence and minimum rental cost for the sequence S_p is

$R(S_p)=\sum_{i=1}^{n} a_{i1}(S_p)\times C_1 + U_2(S_p)\times C_2$,where C_1 and C_2 are the rental cost per unit time of 1st and 2nd machine respectively.

8. Numerical Illustration

Consider 5 jobs, 2 machine flow shop problem with processing time described by triangular fuzzy number as given in the following table and jobs 2, 5 are to be processed as a group job (2, 5). The rental cost per unit time for machines M_1 and M_2 are 4 units and 3 units respectively. Our objective is to obtain optimal schedule to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines, under the rental policy P.

Jobs	Machine M_1	Machine M_2
i	a_{i1}	a_{i2}
1	(7,8,9)	(6,7,8)
2	(12,13,14)	(5,6,7)
3	(8,10,12)	(4,5,6)
4	(10,11,12)	(5,6,7)
5	(9,10,11)	(5,6,8)

(Table 2)

Solution: As per step 1: The A.H.R of processing time of jobs is as follows

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	26/3	23/3
2	41/3	20/3
3	34/3	17/3
4	35/3	20/3
5	32/3	21/3

(Table 3)

As per step 2: Here β = (2,5)

$$A_{\beta 1} = 41/3 + 32/3 - 20/3 = 53/3$$
$$A_{\beta 2} = 20/3 + 21/3 - 20/3 = 21/3$$

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	26/3	23/3
β	53/3	21/3
3	34/3	17/3
4	35/3	20/3

(Table 4)

As per step 4: Using Johnson's method optimal sequence is

S = 1 – β – 4 – 3 i.e. 1 – 2 – 5 – 4 – 3

As per step 5: The In – Out table for the sequence S is as shown in table

Jobs	Machine M_1	Machine M_2
	In- out	In – out
1	(0,0,0) – (7,8,9)	(7,8,9) – (13,15,17)
2	(7,8,9) – (19,21,23)	(19,21,23) – (24,27,30)
5	(19,21,23) – (28,31,34)	(28,31,34) – (33,37,42)
4	(28,31,34) – (38,42,46)	(38,42,46) – (43,48,53)
3	(38,42,46) – (46,52,58)	(46,52,58) – (50,57,64)

(Table 5)

Total elapsed time $t_{n,2}(S)$ = (50,57,64)

As per step 6: The latest time at which machine M_2 is taken on rent

$$U_2(S) = t_{n,2}(S) - L_2(S) = (25,30,36)$$

As per step 7: The utilization time of machine M_2 is

$$U_2(S) = t_{n,2}(S) - L_2(S) = (25,30,36)$$

Bi-objective In – Out table is as follows

Jobs	Machine M_1	Machine M_2
i	In – Out	In – Out

1	(0,0,0) – (7,8,9)	(25,27,28) – (31,34,36)
2	(7,8,9) – (19,21,23)	(31,34,36) – (36,40,43)
5	(19,21,23) – (28,31,34)	(36,40,43) – (41,46,51)
4	(28,31,34) – (38,42,46)	(41,46,51) – (46,52,58)
3	(38,42,46) – (46,52,58)	(46,52,58) – (50,57,64)

(Table 6)

Total Minimum Rental Cost = $R(S) = \sum_{i=1}^{n} a_{i1} \times C_1 + U_2(S) \times C_2 = (259,298,340)$

9. Conclusion

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} a_{i,2}(S)$ on M_2 will, reduce the idle time of all jobs on it. Therefore total rental cost of M_2 will be minimum. Also rental cost of M_1 will always be minimum as idle time of M_1 is always zero. The study may further be extending by introducing the concept of transportation time, Weightage of jobs, Breakdown Interval etc.

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